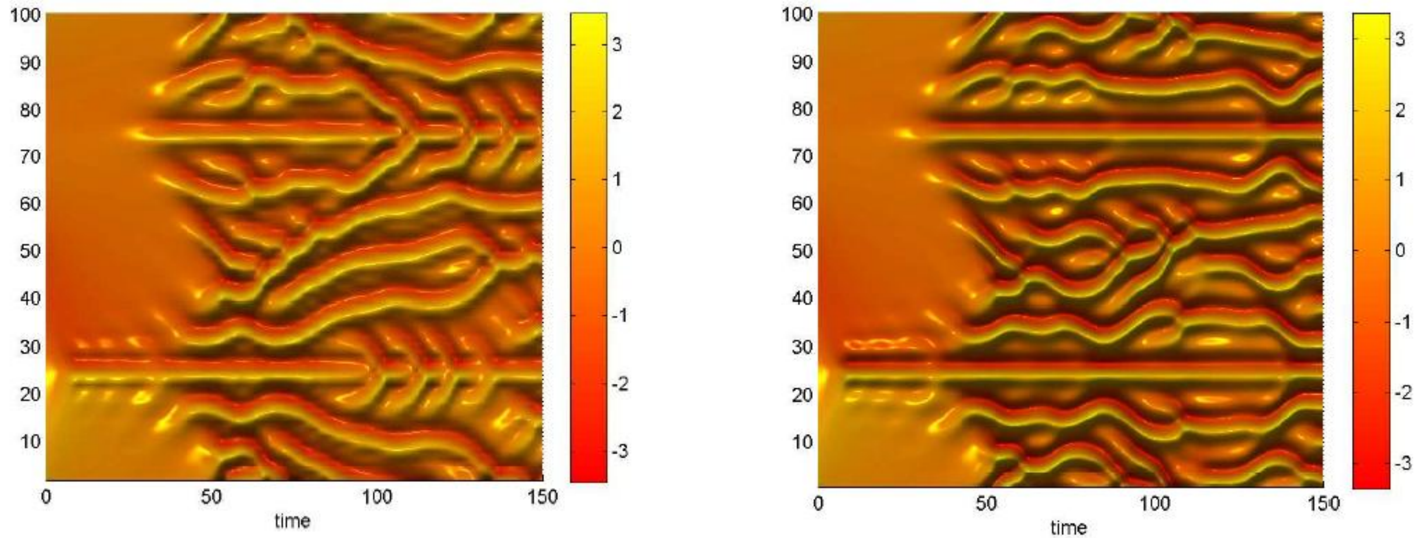
Incorporating Representativity Error in Data Assimilation



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Outline

- Errors of Representativity
- Structure of Representativity Error
- Representativity Error in Data Assimilation
- Summary

Correlated Observation Errors

- Stewart et al (2009, 2012) - Met Office data
- Bormann et al (2010) - ECMWF data

Correlated Observation Errors

- Stewart et al (2009, 2012) - Met Office data
- Bormann et al (2010) - ECMWF data

- Stewart et al (2010, 2012) - SWE system
- Weston (2011) - Met Office system

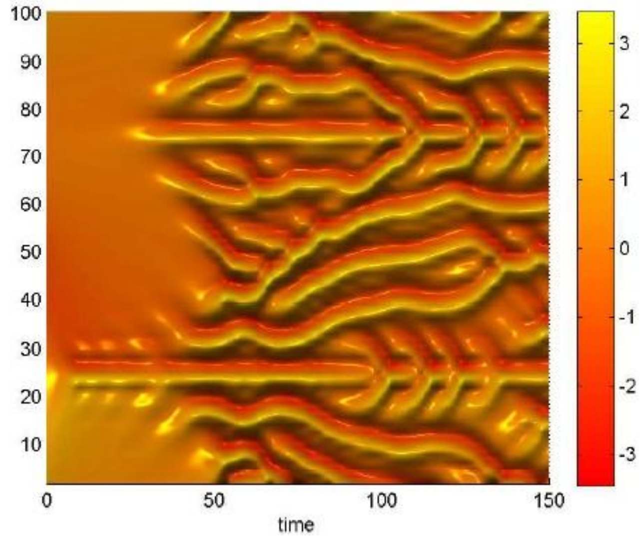
Errors of Representativity

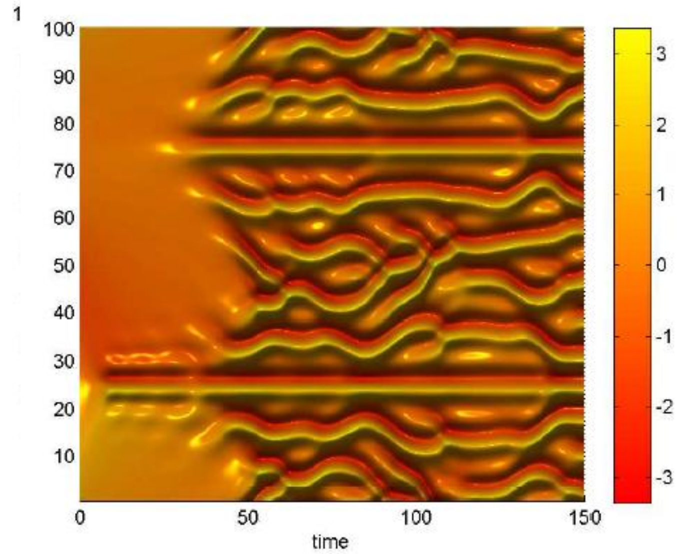
Data assimilation combines observations with a model prediction.

Observations can contain information at smaller scales than the model can resolve.

Errors of representativity are the result of small scale information in observations being incorrectly represented in the model.

Spot the Difference?





Kuromoto-Sivashinsky Equation

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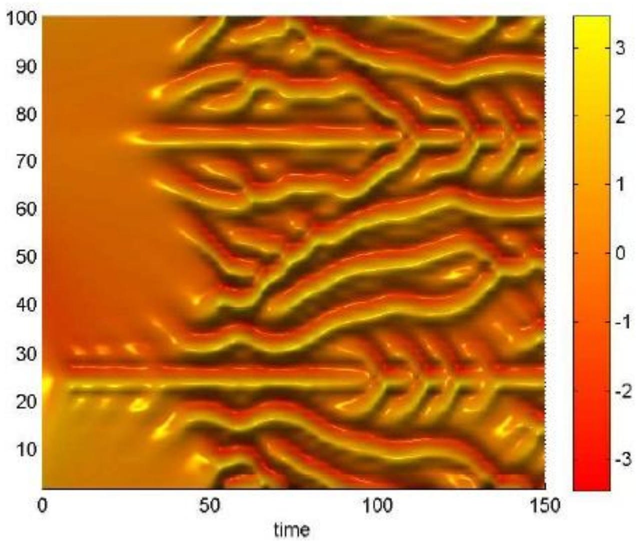
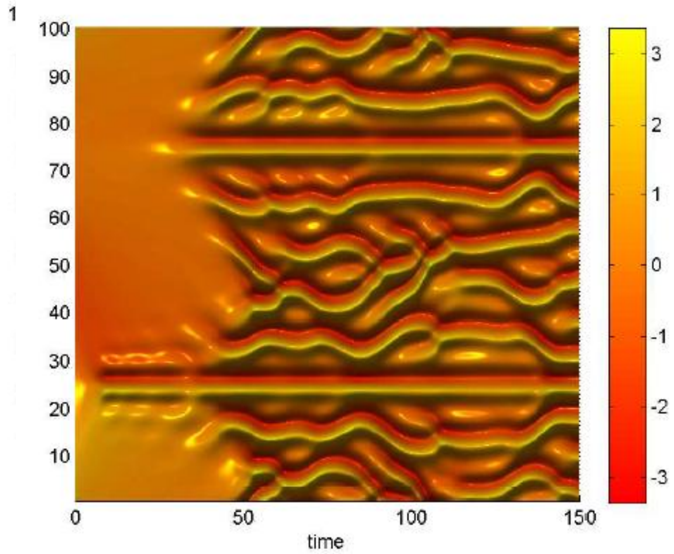
Spot the Difference?



Low resolution



High resolution

Kuromoto-Sivashinsky Equation

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Research Aims

- Investigate the structure and properties of representativity errors

- Incorporate representativity errors in data assimilation

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Structure of Static Representativity Error

Structure of Static Representativity Error

Daley (1993)

Liu & Rabier (2002)

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Structure of Static Representativity Error

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$$\mathbf{y} - \mathcal{H}(\mathbf{x}^{\mathbf{m}}) = \boldsymbol{\epsilon}^{\mathbf{H}}$$

Forward Error

$$\mathbf{y} - \mathcal{H}(\mathbf{x}^{\mathbf{m}}) = \epsilon^{\mathbf{H}}$$

Forward Error

$$\mathbf{y}^{\mathbf{o}} - \mathbf{y} = \epsilon^{\mathbf{I}}$$

Instrument Error

$$\mathbf{y} - \mathcal{H}(\mathbf{x}^{\mathbf{m}}) = \epsilon^{\mathbf{H}}$$

Forward Error

$$\mathbf{y}^{\mathbf{o}} - \mathbf{y} = \epsilon^{\mathbf{I}}$$

Instrument Error

$$\mathbf{y}^{\mathbf{o}} - \mathcal{H}(\mathbf{x}^{\mathbf{m}}) = \epsilon^{\mathbf{H}} + \epsilon^{\mathbf{I}}$$

Observation Error

$$\mathbf{y} - \mathcal{H}(\mathbf{x^m}) = \epsilon^{\mathbf{H}}$$

Forward Error

$$\mathbf{y^o} - \mathbf{y} = \epsilon^{\mathbf{I}}$$

Instrument Error

$$\mathbf{y^o} - \mathcal{H}(\mathbf{x^m}) = \epsilon^{\mathbf{H}} + \epsilon^{\mathbf{I}}$$

Observation Error

$$E[\epsilon^H {\epsilon^H}^T] = \mathbf{R^H} \qquad E[\epsilon^I {\epsilon^I}^T] = \mathbf{R^I}$$

implies

$$\mathbf{R} = \mathbf{R^H} + \mathbf{R^I}$$

Observation Error Covariance

Assumption: Model state is a truncation in spectral space of a high resolution ‘true‘ state .

$$\mathbf{x} = \mathbf{F}\hat{\mathbf{x}} \qquad \mathbf{x}^{\mathrm{m}} = \mathbf{F}^{\mathrm{m}}\hat{\mathbf{x}}^{\mathrm{m}} = \mathbf{F}^{\mathrm{m}}\mathbf{T}\hat{\mathbf{x}}$$

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Assumptions: Model state is a truncation in spectral space of a high resolution ‘true‘ state . Observation operator is linear.

$$\mathbf{x} = \mathbf{F}\hat{\mathbf{x}} \qquad \mathbf{x}^{\mathbf{m}} = \mathbf{F}^{\mathbf{m}}\hat{\mathbf{x}}^{\mathbf{m}} = \mathbf{F}^{\mathbf{m}}\mathbf{T}\hat{\mathbf{x}}$$

then

$$\mathbf{y} = \mathbf{F}_{\mathbf{p}}\mathbf{W}\hat{\mathbf{x}} \qquad \mathcal{H}(\mathbf{x}^{\mathbf{m}}) = \mathbf{F}_{\mathbf{p}}^{\mathbf{m}}\mathbf{W}^{\mathbf{m}}\mathbf{T}\hat{\mathbf{x}}$$

where

$$y(r_o) = \int_{-a\pi}^{a\pi} x(r)w(r - r_o)dr$$

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Therefore:

$$\mathbf{R}^{\mathbf{H}} = (\mathbf{F}_{\mathbf{p}}\mathbf{W} - \mathbf{F}_{\mathbf{p}}^{\mathbf{m}}\mathbf{W}^{\mathbf{m}}\mathbf{T})\hat{\mathbf{S}}(\mathbf{F}_{\mathbf{p}}\mathbf{W} - \mathbf{F}_{\mathbf{p}}^{\mathbf{m}}\mathbf{W}^{\mathbf{m}}\mathbf{T})^*$$

where

$$\hat{\mathbf{S}} = E[\hat{\mathbf{x}}\hat{\mathbf{x}}^*]$$

or equivalently:

$$\hat{\mathbf{S}} = \mathbf{F}^*\mathbf{S}\mathbf{F}\,, \quad \mathbf{S} = E[\mathbf{x}\mathbf{x}^*]$$

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Experiments

Assumption: Model state is a truncation of a high resolution ‘true‘ state .

True states are temperature and humidity from MetO UKV (1.5 km) model. Truncation is x 8 (~ 12 km grid). Observed at all model grid points.

Two cases -

- Case 1: cloudy/ convection
- Case 2: slow tracking deep depression

Data – Case 1

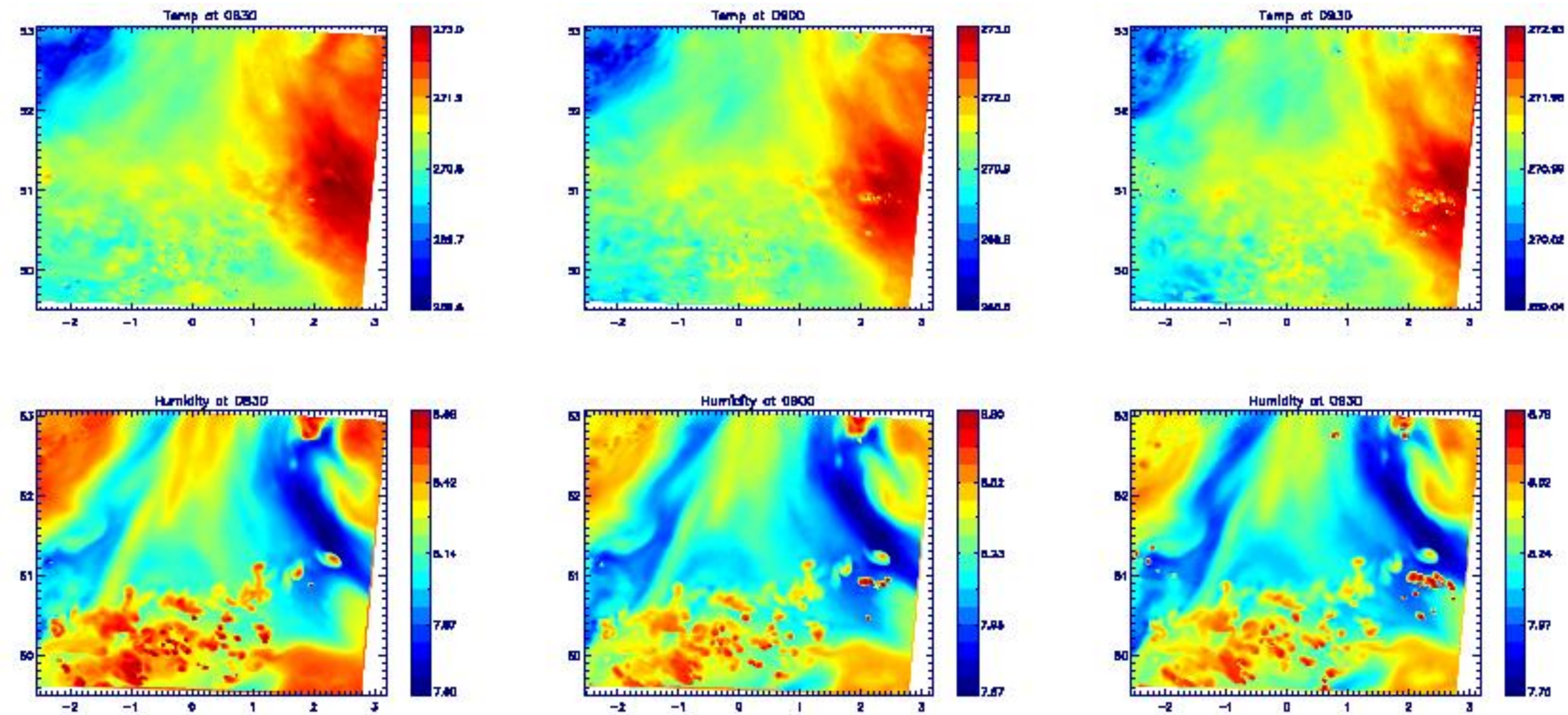

Figure – Temperature and specific humidity profiles at 749 hPa

True Correlation Structure

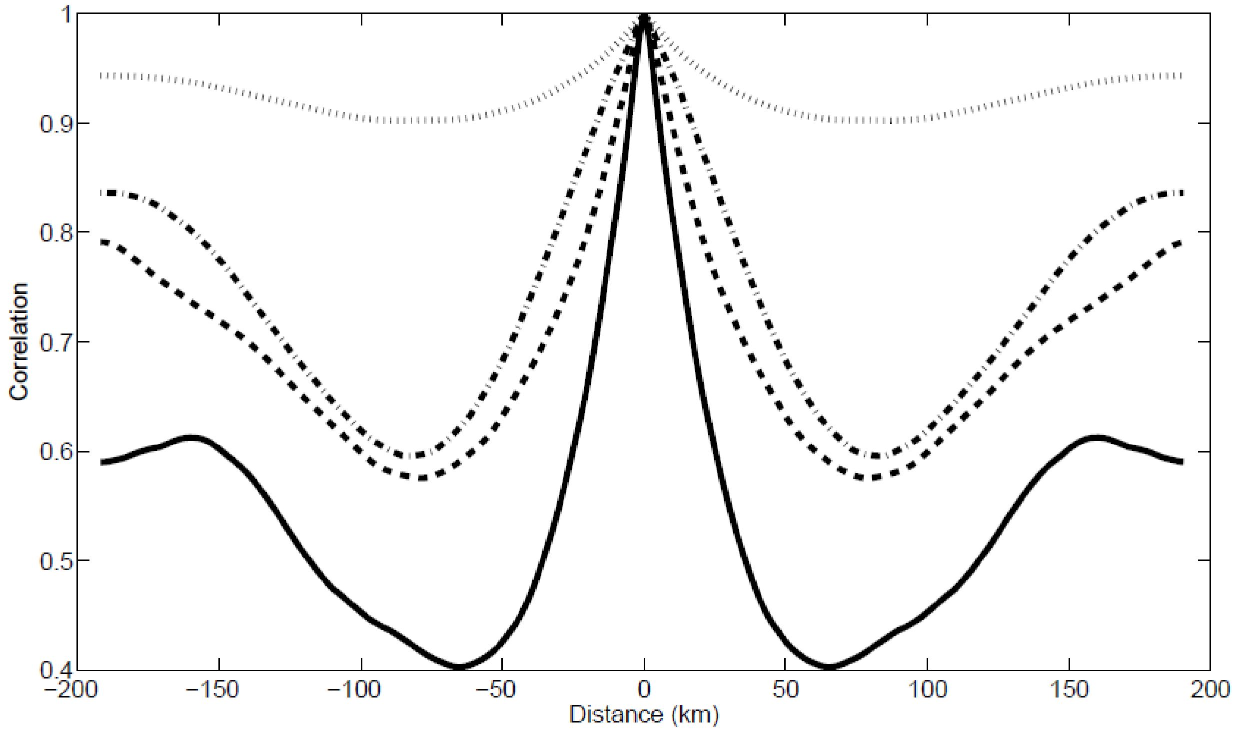

Figure 1. Correlation structure for the true temperature and ln of specific humidity fields at 749hPa. Temperature: Case 1 dotted line, Case 2 dot-dash line. Specific Humidity: Case 1 dashed line, Case 2 solid line

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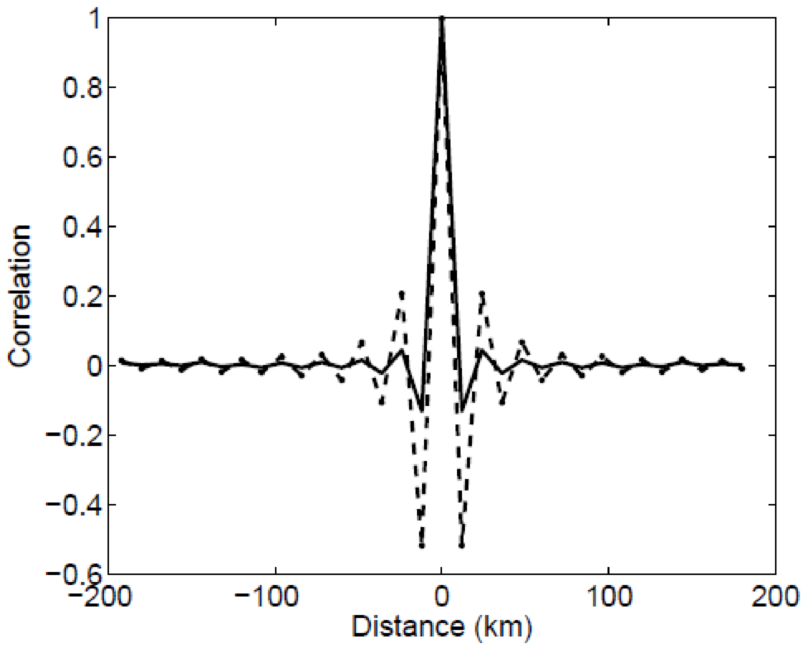
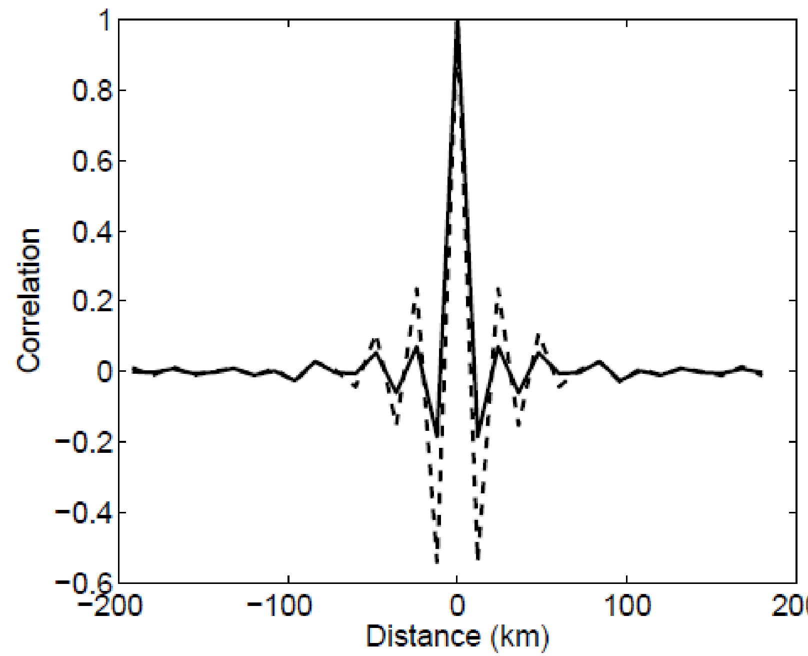
Representativity Error Correlation Structure



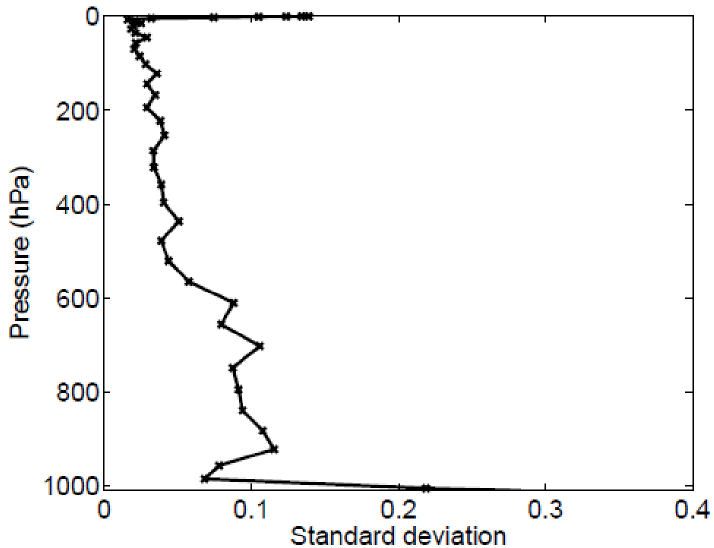
(a) Temperature (K)



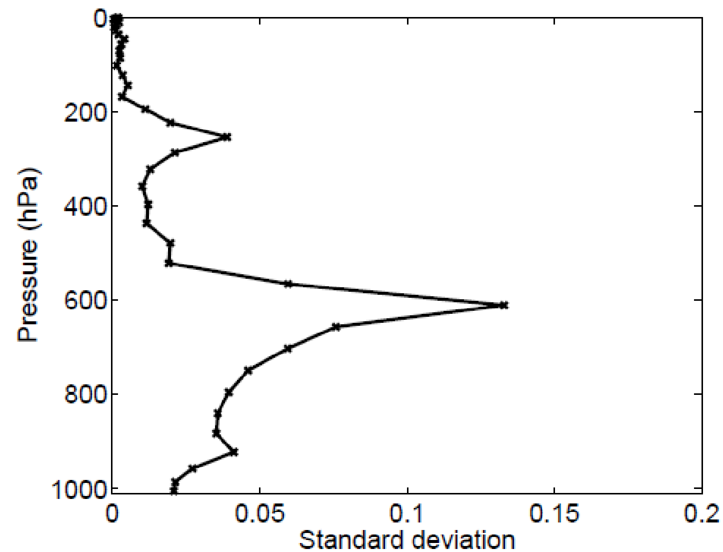
(b) ln(Specific humidity) (kg/kg)

Figure 2. Representativity error correlations between observation center points for Case 1 at 749hPa with truncation to 32 points (12km resolution) with every model grid point observed using direct (solid line) and Gaussian-weighted (dashed line) observations. a) Temperature b) ln(Specific humidity)

Results - Case 1



(a) Temperature (K)



(b) log(Specific humidity) (kg/kg)

Standard Deviations of Representativity Errors

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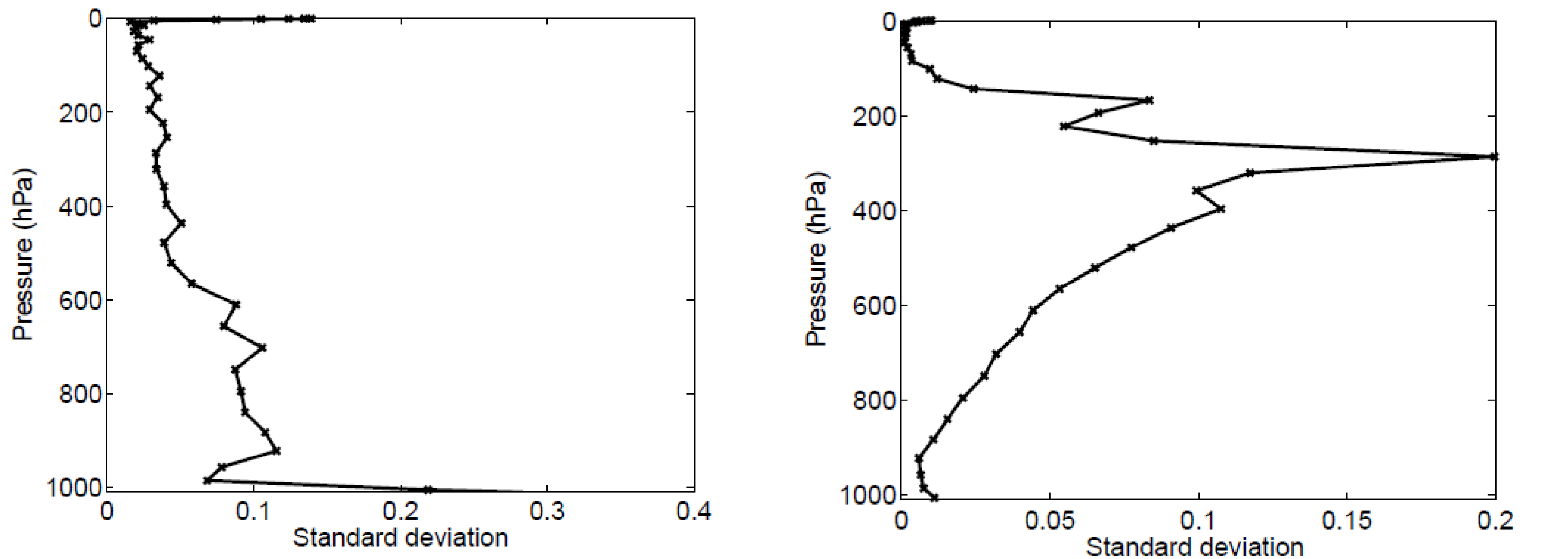
Results - Case 2



(a) Temperature (K)

(b) log(Specific humidity) (kg/kg)

Standard Deviations of Representativity Errors

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Conclusions

Errors of representativity:

- are correlated and state and time dependent;
- are reduced by increasing model resolution or increasing observation length scale;
- vary with height throughout the atmosphere;
- are more significant for humidity than temperature;
- depend only on distance between observations and not the number.

Pocock et al, *U of Reading Maths, Preprint MPS 2012-19*

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Use in Data Assimilation

Difficulties:

- Spectral covariance of truth is unknown
- Operators not invertible
- Error covariances are static

Representativity Error in Data Assimilation

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Representativity Error in Data Assimilation

Desroziers et al (2005)

$$\mathbf{d}^{\mathrm{b}} = \mathbf{y}^{\mathrm{o}} - \mathcal{H}(\mathbf{x}^{\mathrm{b}}) \qquad \mathbf{d}^{\mathrm{a}} = \mathbf{y}^{\mathrm{o}} - \mathcal{H}(\mathbf{x}^{\mathrm{a}})$$

$$\mathbf{d}^{\mathrm{b}} = \mathbf{y}^{\mathrm{o}} - \mathcal{H}(\mathbf{x}^{\mathrm{b}}) \qquad \mathbf{d}^{\mathrm{a}} = \mathbf{y}^{\mathrm{o}} - \mathcal{H}(\mathbf{x}^{\mathrm{a}})$$

$$\begin{aligned} E[\mathbf{d}^{\mathrm{a}}\mathbf{d}^{\mathrm{b}T}] &= \mathbf{R}(\mathbf{HBH}^T + \mathbf{R})^{-1} E[\mathbf{d}^{\mathrm{b}}\mathbf{d}^{\mathrm{b}T}] \\ &= \mathbf{R}(\mathbf{HBH}^T + \mathbf{R})^{-1}(\mathbf{HBH}^T + \mathbf{R}) \\ &= \mathbf{R} \end{aligned}$$

$$\mathbf{d}^{\mathrm{b}} = \mathbf{y}^{\mathrm{o}} - \mathcal{H}(\mathbf{x}^{\mathrm{b}}) \qquad \mathbf{d}^{\mathrm{a}} = \mathbf{y}^{\mathrm{o}} - \mathcal{H}(\mathbf{x}^{\mathrm{a}})$$

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This is valid if B and R used to find the analysis are exact, although, if not, a reasonable estimate of R may be obtained. The calculated R must then be symmetrized.

Procedure

- Select initial R
- Run ETKF and gather samples of d^b and d^a
- Compute $E[d^a \, d^{b*}]$
- Symmetrize and localize to obtain new estimate for R
- Repeat steps of ETKF using samples from rolling window of length N_s to update R

Experiments

Add errors to observations from given distribution with known SOAR covariance R.

- Assume incorrect R_I = diagonal at t= 0. Recover true covariance.

- Allow length scale in covariance to vary slowly. Recover time-varying covariance.

Pocock-Waller, *U of Reading Maths PhD 2013*

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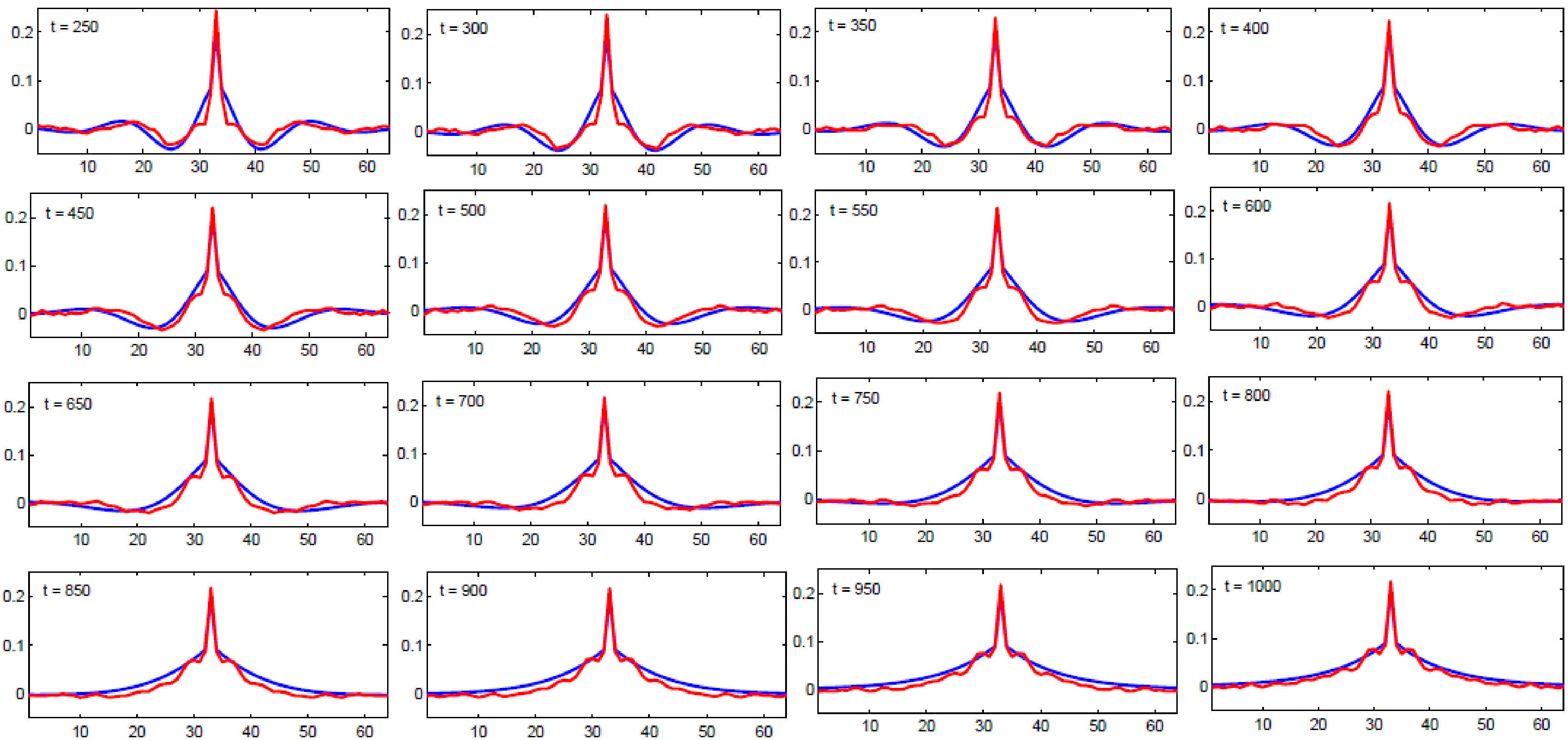
Results



Rows of the true and estimated correlation matrices

Kuromoto-Sivashinsky Equation

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Conclusions

- Time-varying observation error covariance matrices can be estimated using an ETKF

- Including the estimated observation error covariances in the data assimilation scheme can improve the analysis

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Future

Many more challenges left!