How Warm is it Getting?

The Determination of a Trend in a Multi-Scale Problem

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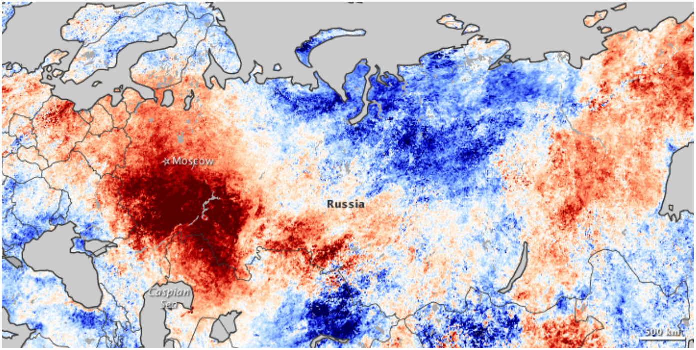
February 19, 2013

Local Warming: Moscow's Summer 2010 Temperatures

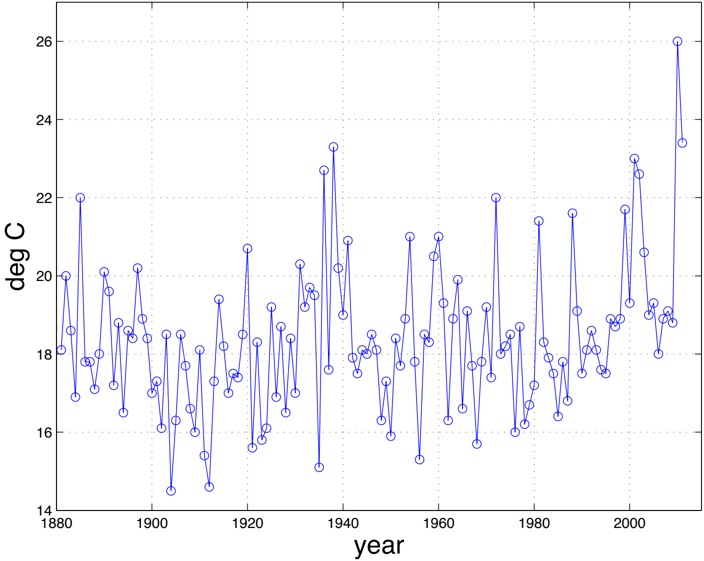
Land surface temp anomalies (Satellite), for July20-27, 2010, compared to July20-27 (2000-2008).



Color Range: -12^{o}C to 12^{o}C

Picture, courtesy of NASA/Goddard/Earth Observatory.

Moscow's Summer Temperatures, 1881-2011



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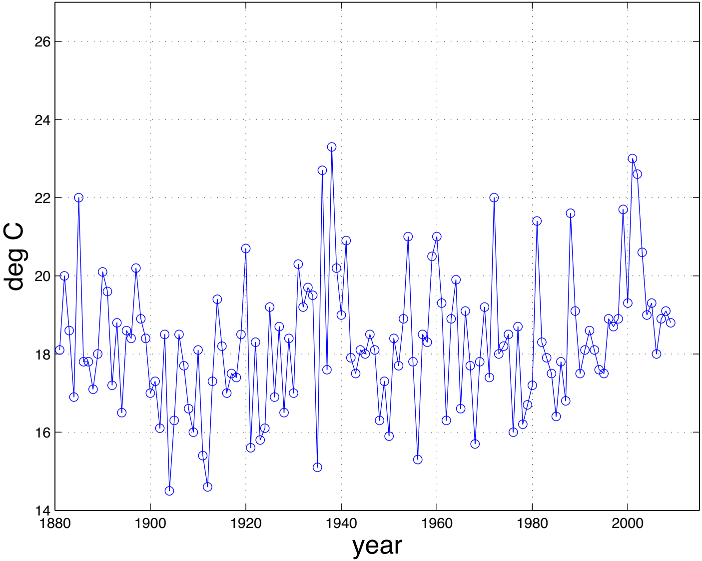

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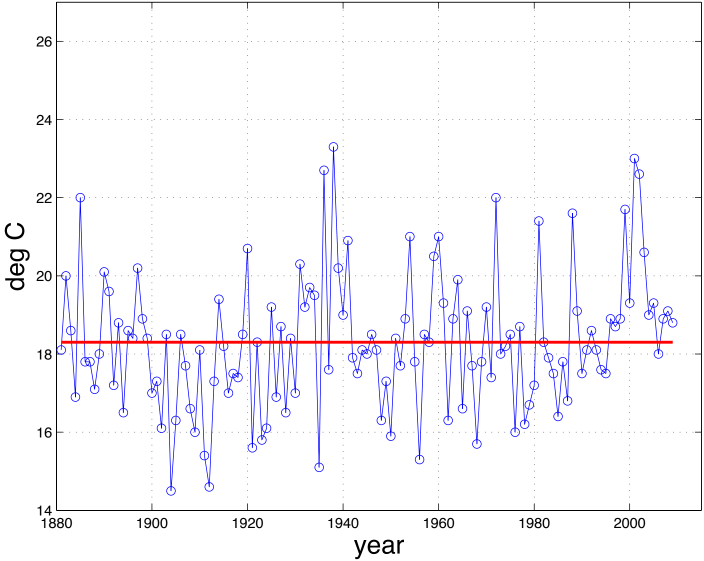

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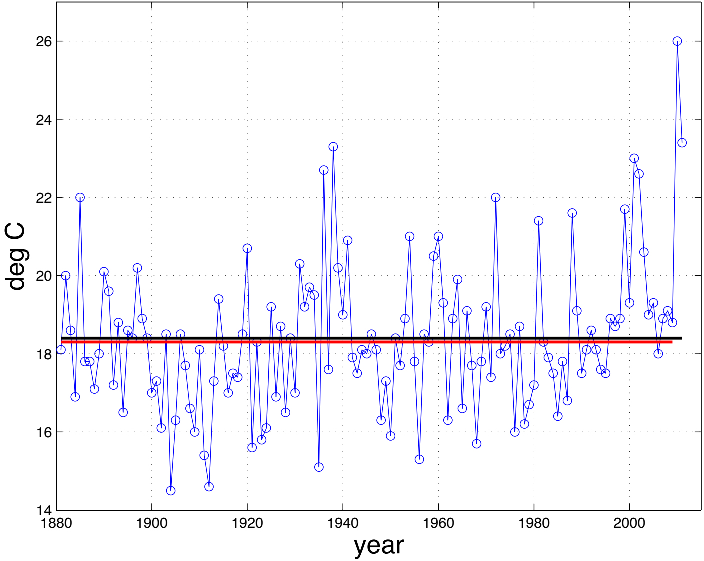
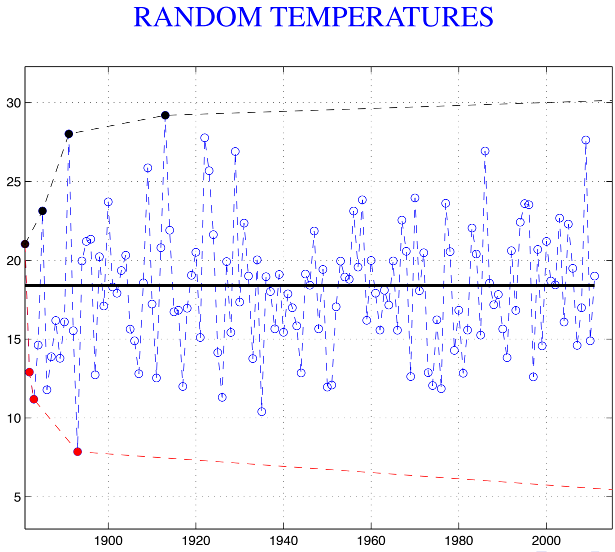
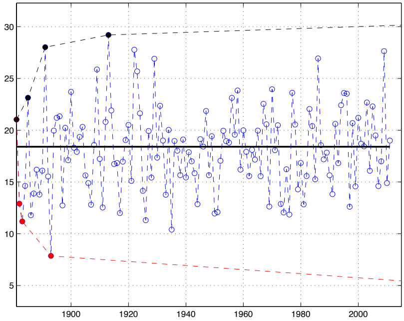


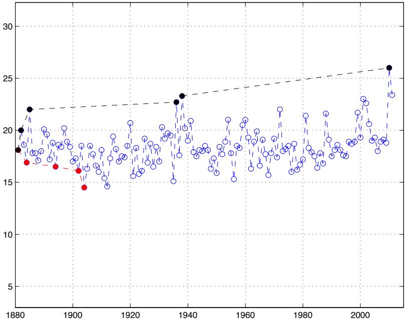
A Mathematical Fact, Applicable to Extreme Temperatures



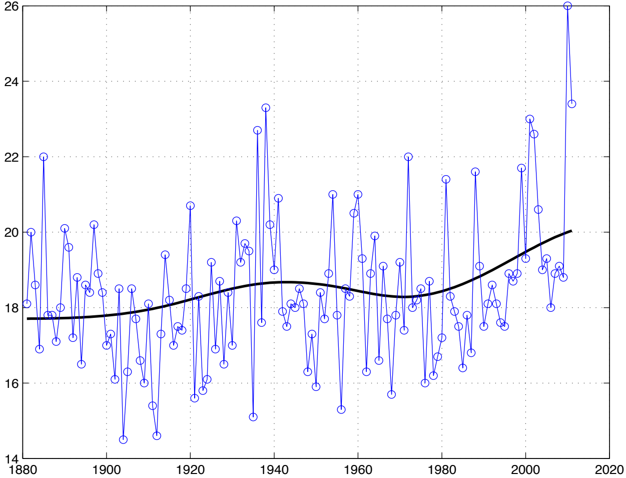
RANDOM TEMPERATURES



MOSCOW TEMPERATURES



Something Must Account for Changing Mean



Increase of Extreme Events in a Warming World (PNAS **44**, 2011), by Rahmstorf and Coumou.

2010 Moscow Hot Summer: Antropogenic Source?

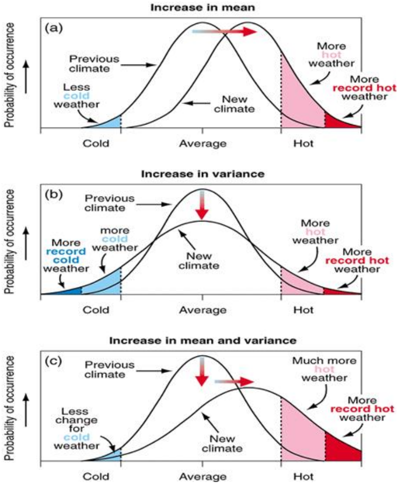


Figure courtesy of Rahmstorf and Coumou. Can be found at realclimate.org.

The Trend Problem:

Define a set of simple universal rules with which to compute an underlying *tendency*, given a finite (non-stationary/multi-scale) data set.

Joint work with
Shankar Venkataramani (U. Arizona)
H. Flaschka (U. Arizona) and
D. Comeau (U. Arizona)

GULF OF MEXICO RESEARCH INITIATIVE

National Science Foundation
WHERE DISCOVERIES BEGIN

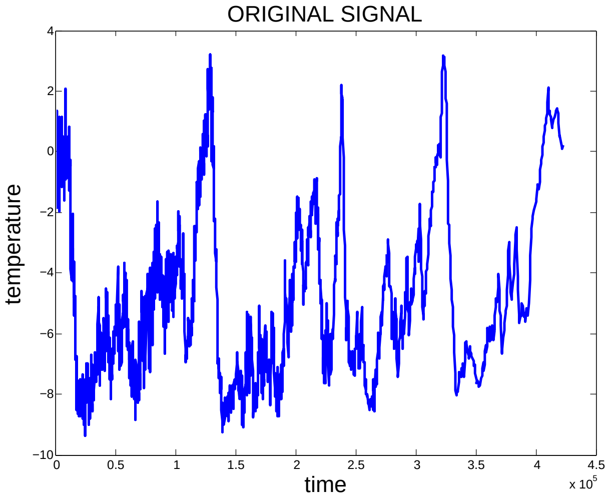
Problems That Critically Depend on a Trend Calculation

- Global warming (sun radiation, CO_2 averages, global temperature estimates).
- Mean sea level (land ice melt and its effect on sea rise).
- Variability of local weather.
- Glacial ice packing.
- Long-term ocean sea surface temps (SST) data: PCA has an ENSO-line signal, not in ocean models.[1]

Other applications: trends in hydrogeology, econometrics, etc.

[1] Robert Miller (COAS/ORST), private communication.

A Climate Signal...



Vostok Ice Core data, Temperature

The Tendency, Defined

Given a finite-time time series $Y(i), \quad i = 1, 2, \ldots, N,$

The tendency $T(i)$ is a time series

- $T(i) := B^D + \{R^j(i)\}_S$, $i = 1, \ldots, N$. B^D is a constant, $\{R^j(i)\}_S$ is a function made up of a combination of S *rotations*.

- *The histogram of $Y(i) - T(i)$ should be nearly symmetric, and $var[T(i)] \leq var[Y(i)]$. (If $T(i) \neq Y(i)$).*
- *Low complexity of $T(i)$, measured by the Hellinger distance: $Hell[Y(i) - T(i)]$ small.*
- if $Y(i)$ is monotonic, $T(i)$ is monotonic.
- if $Y(i)$ is a constant, $T(i) = Y(i)$.
- if $Y(i)$ is stationary ($N \to \infty$), $T(i) = m$, the median.

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General Procedure:

- Find a decomposition $Y(i) = B^D + \sum_{j=1}^{D} R^j(i)$
- Apply tendency criteria to pick a combination of R^j to form $T(i) := B^D + \{R^j(i)\}_S$, $i = 1, ..., N$.

The choice of decomposition is motivated by the

- Be non-parametric.
- Ability to handle multi-scale nature of a signal.
- Be lossless.

The Decomposition

The Intrinsic Time Decomposition (ITD)

Given a sequence of real numbers $\{Y(i)\}_{i=1}^{N}$,

$$Y(i) = B^D + \sum_{j=1}^{D} R^j(i)$$

where

$$\begin{aligned} B^j(i) &= B^{j+1}(i) + R^{j+1}(i), \quad j = 0, \ldots, D, \\ &\text{and} \\ B^0(i) : &= Y(i). \end{aligned}$$

B^j are called *BASELINES*, and R^j are called *ROTATIONS*.

Frei and Osorio, Proc. Roy. Soc. London, (2006).

The Intrinsic Time Decomposition (ITD)

Baseline Construction:

- Identify extremas $Y_k := Y(\tau_k)$ and nodes τ_k.
- Construct knots B_k,

$$
\begin{aligned}
B_{k+1} &= \frac{1}{2}\left[Y_k + \frac{(\tau_{k+1}-\tau_k)}{(\tau_{k+2}-\tau_k)}(Y_{k+2}-Y_k)\right] \\
&+ \frac{1}{2}Y_{k+1}.
\end{aligned}
$$

In the interval $i \in (\tau_k, \tau_{k+1}]$, between successive extrema,

$$
\begin{aligned}
B(i) &= B_k + \frac{(B_{k+1}-B_k)}{(Y_{k+1}-Y_k)}(Y(i)-Y_k), \\
R(i) &= Y(i) - B(i).
\end{aligned}
$$

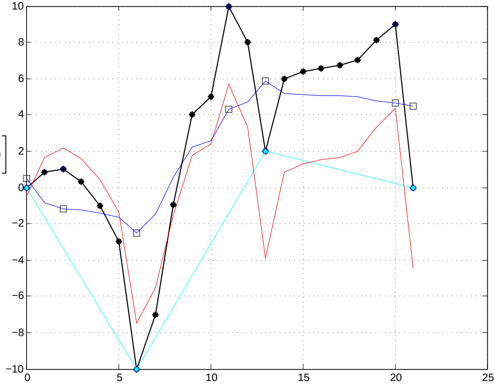


Figure: Signal Y, Rotation R, Baseline B

The Intrinsic Time Decomposition (ITD)

General Case: Set $B^0(i) = Y(i)$, $i = 1, 2, ..., , N$.
For $j = 0, ..., D-1$:

$R^{j+1}(i) = B^j(i) - B^{j+1}(i)$.

- Identify extremas $B^j_k := B^j(\tau^j_k)$ and nodes τ^j_k.
- Construct knots B^{j+1}_k,

$$B^{j+1}_{k+1} := B^{j+1}(\tau^{j+1}_{k+1}) = \frac{1}{2}\left[B^j_k + \frac{(\tau^j_{k+1} - \tau^j_k)}{(\tau^j_{k+2} - \tau^j_k)}(B^j_{k+2} - B^j_k)\right] + \frac{1}{2}B^j_{k+1}.$$

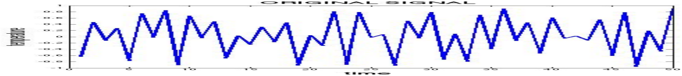
In the interval $i \in (\tau^{j+1}_k, \tau^{j+1}_{k+1}]$, between successive extrema,

$$B^{j+1}(i) = B^j_k + \frac{(B^j_{k+1} - B^j_k)}{(B^j_{k+1} - B^j_k)}(B^j(i) - B^j_k),$$

Intuition

Define the *All Extrema Random Signal*

$Y(i) = (-1)^i |z_i|, \quad i = 1, 2, ..., N$, with z_i a sample from $\mathcal{N}(0, \sigma)$



In this case

$$B(i) = B_k = \frac{1}{4}(Y_{k-1} + 2Y_k + Y_{k+1}).$$

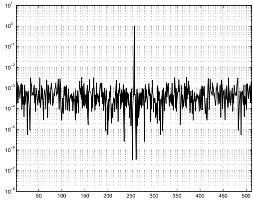
and

$$R(i) = R_k = Y_k - B_k = -\frac{1}{2}(Y_{k-1} - 2Y_k + Y_{k+1}).$$

Even if extremas are not equally-spaced:,

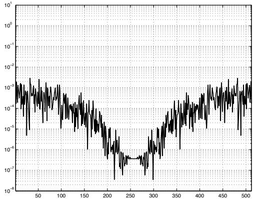
- *if* $B = \mathcal{L}Y$, *then* $R = (1 - \mathcal{L})Y$,
- $B^{j+1} = \mathcal{L}^j B^j$
- $R^{j+1} = (1 - \mathcal{L}^j) B^j$.

The Fourier transform $\hat{Y}$:

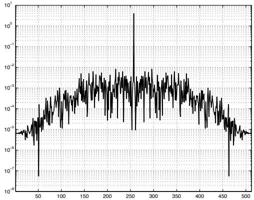


$\omega = 2\pi\nu/N$, and $0 \leq \nu \leq N/2$, the integer frequency.

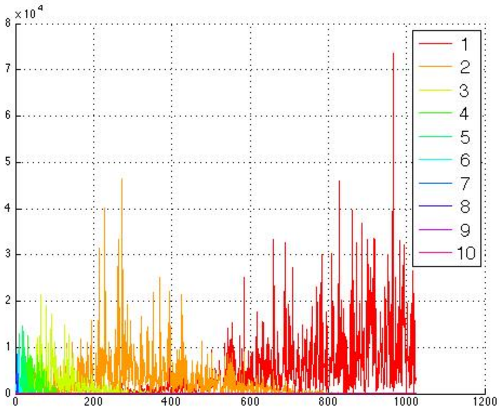
The baseline: $\frac{\hat{B}}{\hat{Y}} = \frac{1}{2}(1+\cos\omega)$



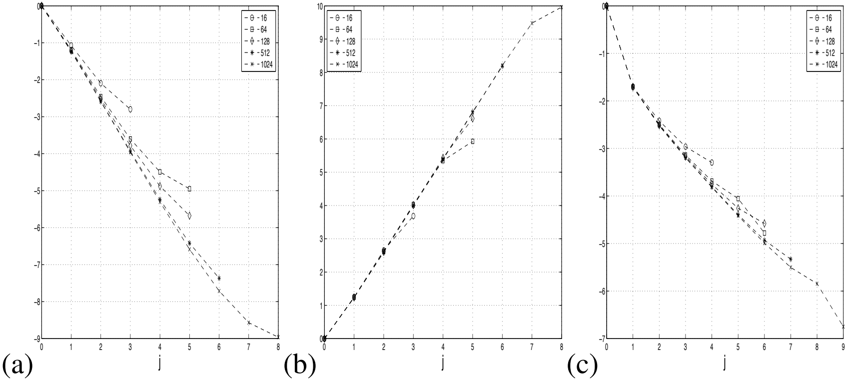
The rotation: $\frac{\hat{R}}{\hat{Y}} = \frac{1}{2}(1-\cos\omega)$



Spectrum of the Rotations



For All Extrema Random Signal $Y = (-1)^i |z_i|, \quad z_i$ from $\mathscr{U}(\sigma = 4)$
Logarithm, base 2, as a function of ITD level j:



(a) Extremas$/N$; (b) Extrema spacing; (c) $\ell_2(B^j)/\ell_2(Y)$

Ensemble averages (50,000 realizations).

Self Similar Spectrum and Extremas

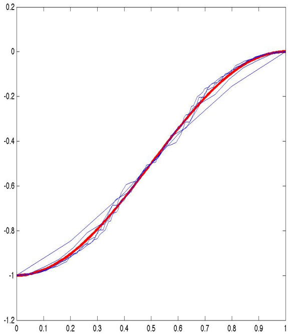
Define $\mathscr{E}[B^j] := \{S^j, b^j\}$.

$\{S^j\}_1^{n_j}$ be locations of extrema of baselines, with values b^j.

In ITD: $\{S^{j+1}, b^{j+1}\} = \mathscr{E}[(\mathbb{I} + M^j)b^j]$.

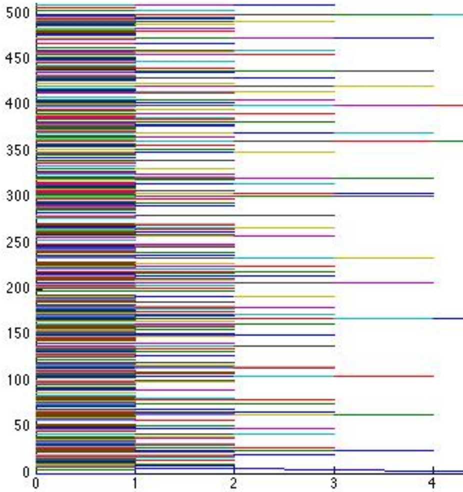
M^j is a diffusion matrix.

- $\mathrm{Sp}(\mathbb{I} + M^j)$ real, $\in [0,1]$: $\lambda_k^j = \cos^2(\pi k/n)$,
- 1 is an eigenvalue corresponding to the right eigenvector consisting of all ones, and 0 is an eigenvalue corresponding to the right eigenvector given by $x_k = (-1)^k$. (Proof is by a Perron-Frobenius type argument).

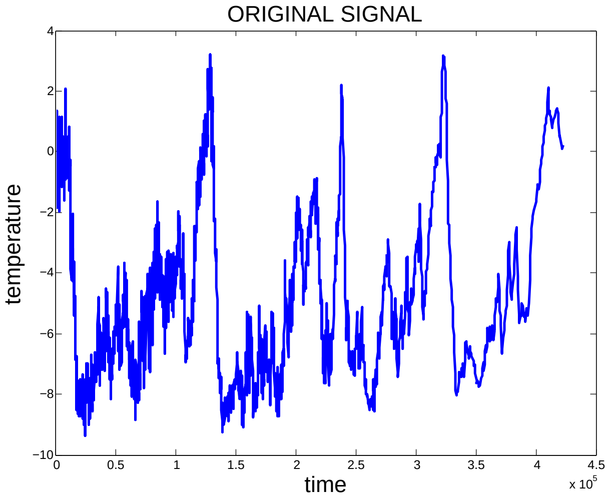


Estimate of probability of extremas disappearing can be found:

- Extrema disappear independently from neighbors.
- Obtain Poisson process for evolution of the sets S^j.

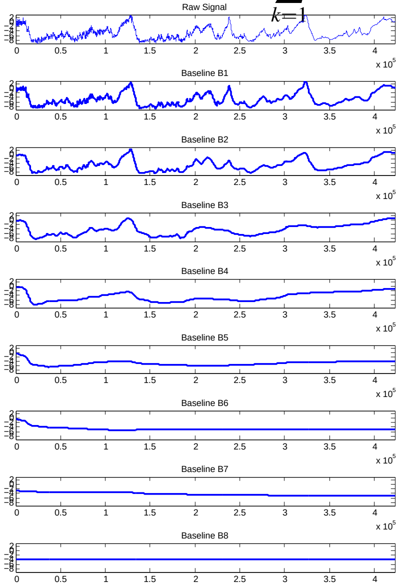


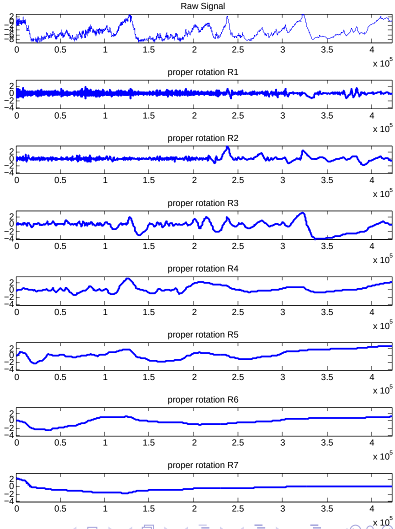
Example Calculation



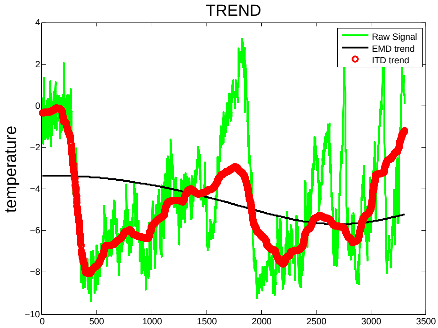
Vostok Ice Core data, Temperature

$$Y(i) = B^D + \sum_{k=1}^{D} R^k(i), \quad B^{j+1}(t) + R^{j+1}(i) = B^j(i).$$

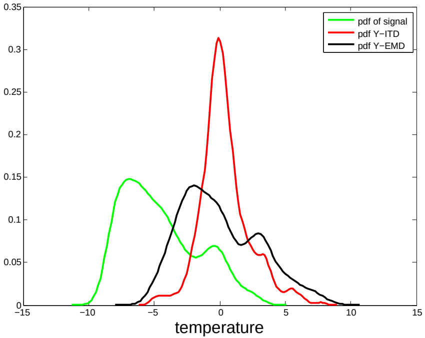




The Tendency $T(i)$, the EMD, and the Vostok signal $Y(i)$



Time Series



The Histograms

Finding the Tendency

- Find ITD:

$$Y(i) = B^D + \sum_{J=1}^{D} R^j(i),$$

$$B^j(i) = B^{j+1}(i) + R^{j+1}(i)$$

- Find Tendency (picking k^* baseline)

$$T(i) := B^{k^*}(i)$$

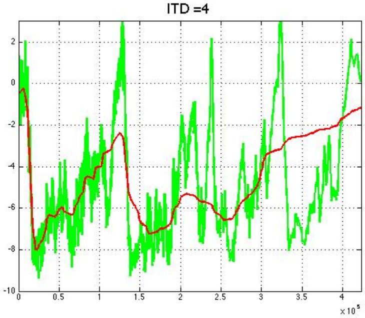
- Find Tendency (choosing k^* baseline)

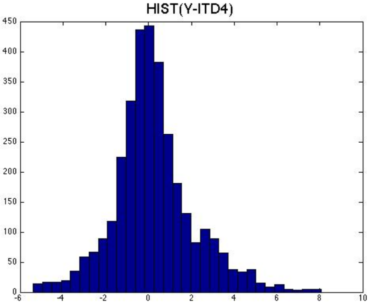
$$T(i) := B^{k^*}(i)$$

- The "ABSISSA" information:
  - For $j = 1, .., D$ compute $H^j :=$ histogram$(Y(i) - B^j(i))$
  - Determine "symmetry" of H^j: via percentiles.
  - Candidates have a symmetric unimodal distribution with variance, smaller than var$Y$.
- The "ORDINATE" information:
  - Compute matrix corr(B^j).
  - Determine B^{k^*}. Of the set chosen in the Absissa selection, choose $j = k^*$ corresponding to first minima in corr$_{j,j+1}$

can get simpler $T(i)$ by maximizing Hell$(T - R^{j \geq k^*})$.

ABSISSA INFORMATION





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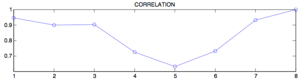
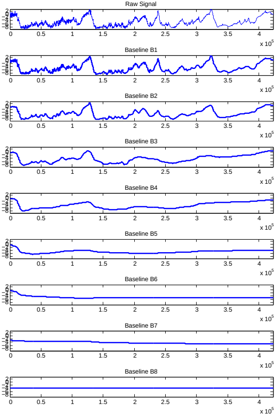
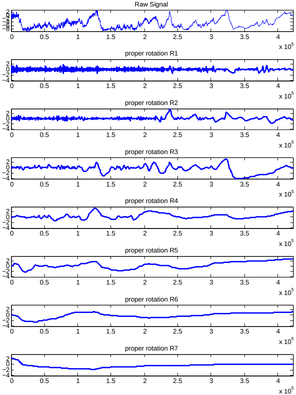
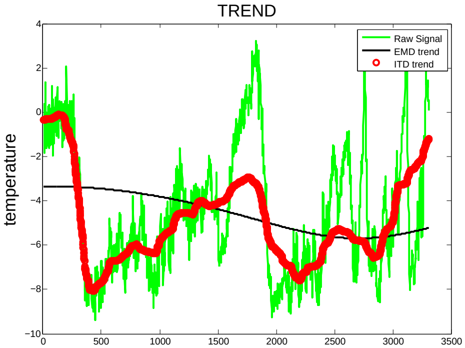
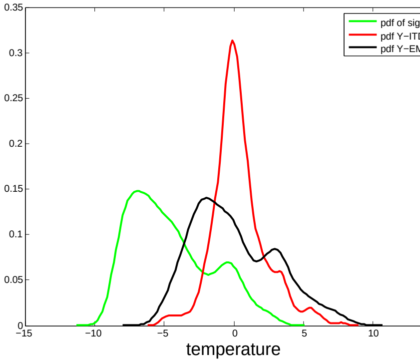






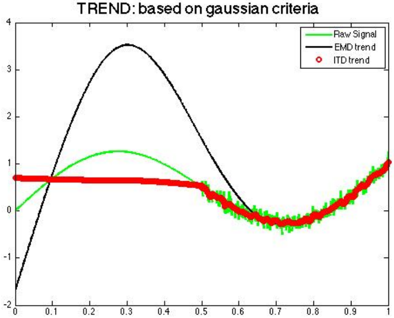


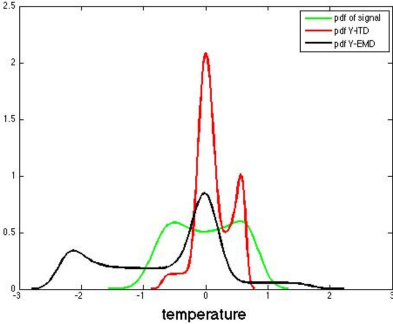


Time Series

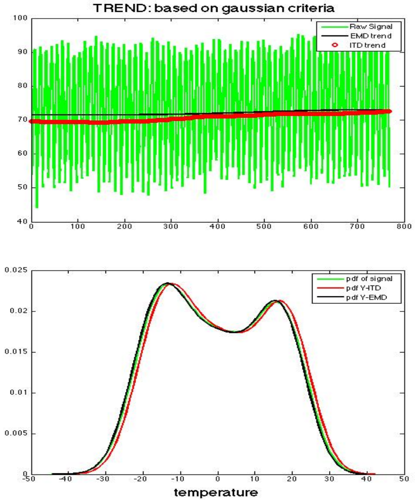
The Histograms

The Composite Case





When There's No Single Trend:



daily temperature data, SW Arizona.

2010 Moscow Hot Summer: Antropogenic Source?

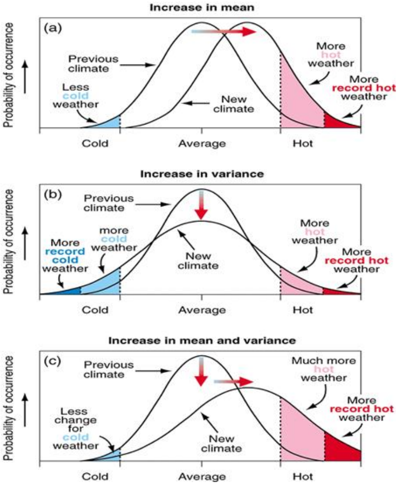
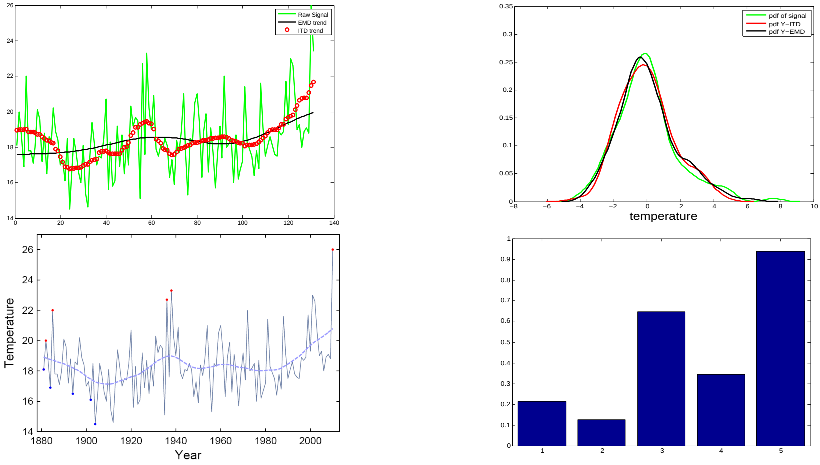


Figure courtesy of Rahmstorf and Coumou. Can be found at realclimate.org.

Analysis of the Moscow Data

Our analysis confirms that Coumou and Rahmstorf's guess that the mean temperature increased, but not its variance:



Other Applications

- 2D image processing?
- Generates a compact **surrogate model** of the form

$$dX_t = f(X_t, t)dt + \sigma dW_t.$$

  - $T(i)$ is the cummulant of the drift term $f(\cdot)$.
  - Estimate σ from hist$(Y - T)$, construct suitable noise process for the diffusion term.

Further Information

Juan M. Restrepo

http://www.physics.arizona.edu/∼restrepo

Uncertainty Quantification Group

http://www.physics.arizona.edu/∼restrepo/UQ/UQ.html